

# Complex Permittivity of Sodium Chloride Solutions at Microwave Frequencies

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The complex permittivity of aqueous solutions at 20 °C has been measured at concentrations between 0.001 and 5 mol/L and over a frequency range 0.13–20 GHz. The results were combined with literature values to derive empirical equations to predict the dielectric behavior of sodium chloride solutions between 0 and 5 mol/L and 5 °C–35 °C. Bioelectromagnetics 28:264–274, 2007. © 2007 Wiley-Liss, Inc.

**Key words:** permittivity; sodium chloride solutions; microwave frequencies

## INTRODUCTION

Biological tissues typically have a high water content and a significant concentration of ions. At frequencies above about 100 MHz the interaction of microwaves with biological material is almost exclusively dependent on the aqueous and ionic content and it is therefore very useful to have accurate values of the complex permittivity of salt solution at these frequencies. In addition, NaCl in aqueous solution is often used as a standard liquid in experiments to measure the dielectric properties of biological material.

So far, it has been common to use the static permittivity values of NaCl, which were published by Stogryn [1971]. However, these values were obtained by extrapolation of the measurements of Lane and Saxon [1952] which were made at frequencies of 9.3, 24.2, and 48.4 GHz and this can involve a significant error, as shown by Grant [1957]. For example, if the extrapolation is carried out using a Debye relationship (which is what Stogryn used) instead of a Cole–Cole relationship (which the present work shows to exist) the error in deducing the static permittivity would be around 5% for a 3 mol/L solution and greater at higher concentrations. Moreover, there is considerable error in the extrapolation process itself, and this is eliminated by making a direct measurement at frequencies around 100–200 MHz.

Apart from the work of Nortemann et al. [1997] and Büchner et al. [1999] which had a low frequency limit of 20 and 200 MHz respectively, there has been little previous work on the dielectric properties of NaCl solutions at these frequencies. At higher frequencies there are many previously published data, and these have been incorporated with the present measurements to obtain the various dielectric parameters and to derive the two empirical relationships referred to above.

## MATERIALS AND METHODS

The NaCl solutions were prepared by volume diluting standard solutions using deionised water to provide a range of concentrations. Solutions in the range 0.001–0.01 mol/L were produced by dilution of 0.01 mol/L standard solution (Fluka cat no 24886, chloride ion standard solution) using water (Fluka 17749, “for analytical purposes”). Solutions in the range 0.01–0.1 mol/L were produced by dilution of 0.1 mol/L standard solution (Fluka cat no 24887) using water (Fluka 17749). Solutions in the range 0.1–1 mol/L were produced by dilution of 1.0 mol/L standard solution (Fluka 71394) using water (Fluka 17749). Solutions above 1.0 mol/L concentration were produced by single step dilution of a 5.0 mol/L solution where the raw material used for preparation of 1000 ml of this solution was sodium chloride (Fluka 31434, puriss, ACS reagent, >99.8%) (58.44 g) and water (Fluka 17749). The accuracy of mass measurement was to 0.01 g in 58.44 g that of volume was to 0.1 ml in 1000 ml.

Twenty samples were prepared ranging from 0.001 to 5 mol/L. The solutions were kept in clean containers avoiding contamination with any impurities and minimal exposure to air. The containers were kept in a water bath to reach a steady 20 °C temperature.

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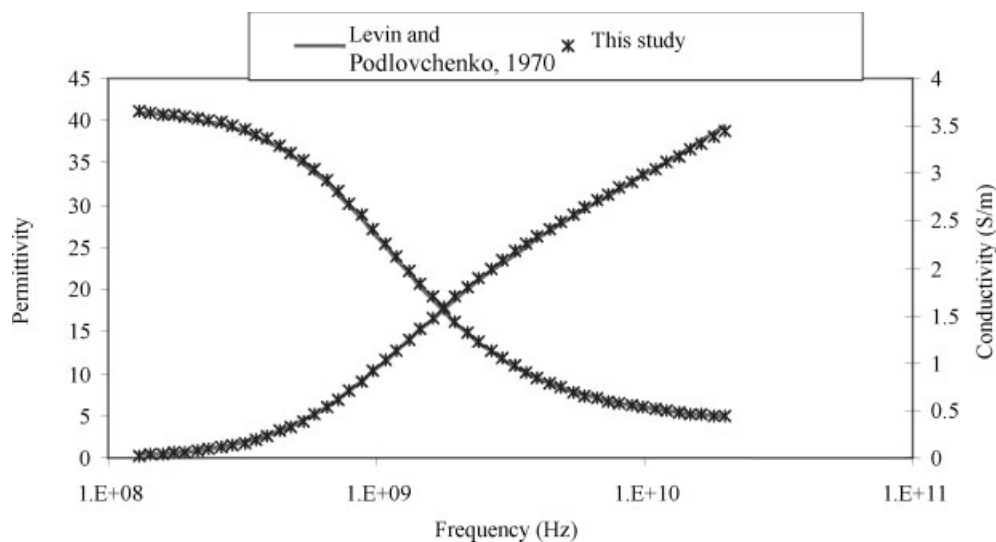


Fig. 1. Measured dielectric properties of ethanediol at 20 °C compared with Levin and Podlovchenko [1970].

Dielectric measurements were carried out using an open-ended coaxial probe (2.98 mm) and a computer controlled network analyzer (HP 8720). The dielectric properties were obtained from  $S_{11}$  parameters using an admittance model described in Gabriel et al. [1994]. This procedure was shown to be quite suitable for dielectric measurement of liquids. When good laboratory practice is adhered to, in particular strict temperature control, the stated accuracy of this procedure is between 1% and 2% when assessed by repeat measurement on standard liquids of well-known dielectric property and the reproducibility is better than 1% of the average values in most frequencies. These figures were confirmed in this study through an uncertainty analysis mentioned in the next section.

### Measurement Uncertainty

The measurement uncertainty analysis was carried out by performing repeated dielectric measurements on a reference liquid: ethanediol; this is a polar

liquid with a static permittivity about half of pure water (Fig. 1), its dielectric properties were taken from Levin and Podlovchenko [1970]. This enables us to calculate random and systematic errors from the repeatability and comparison of measured and literature values. These are then combined with other applicable uncertainty elements to give the total combined uncertainty (Table 1): details of the procedure are published elsewhere [Gabriel and Peyman, 2006].

The measurement uncertainty calculated using ethanediol is fairly typical of that observed with other reference liquids. For example, we obtain similar results using formamide and dimethylene sulphoxide. This is reinforced by the NaCl data reported in this paper, which compare well with Büchner et al. [1999].

No problems were encountered in the measurement of NaCl solution of low and moderate concentrations. However, at high concentration it became increasingly difficult to measure the permittivity in the presence of very high conductivities. Consequently,

TABLE 1. Experimentally Determined Uncertainty Elements and Total Combined Uncertainty for the Permittivity ( $\epsilon'$ ) and Conductivity ( $\sigma$ ) Across the Frequency Range

Frequency Region	Random error (repeatability)		Systematic error		Drift of network analyser		Total combined uncertainty	
	$\% \Delta \epsilon'_{\text{rep}}$	$\% \Delta \sigma_{\text{rep}}$	$\% \Delta \epsilon'_{\text{sys}}$	$\% \Delta \sigma_{\text{sys}}$	$\% \Delta \epsilon'_d$	$\% \Delta \sigma_d$	$\% \Delta \epsilon'$	$\% \Delta \sigma$
100–300 MHz	0.35	1.72	0.41	1.90	0.11	0.10	0.42	2.04
>300 MHz–10 GHz	0.26	0.30	0.81	0.99	0.07	0.24	0.53	0.66
>10–20 GHz	0.09	0.30	0.19	0.80	0.16	0.17	0.17	0.56

Data based on the measurement of the dielectric properties of ethanediol at 20 °C. Weighting factors, that take into account the probability distribution, are applied to the uncertainty elements prior to calculating the total uncertainty as the square root of the sum of the squares of the weighted uncertainty elements (The methodology was taken from NIST 1994 and GUM 1995).

we experienced some instability in the measurement of the permittivity at frequencies below a few hundred megahertz.

**Data Analysis**

The measured dielectric data for all NaCl concentrations were fitted to the following Cole–Cole model

$$\hat{\epsilon} = \epsilon_{\infty} + \frac{\epsilon_s - \epsilon_{\infty}}{1 + (j\omega\tau)^{(1-\alpha)}} + \frac{\sigma_i}{j\omega\epsilon_0} \quad (1)$$

where,  $\epsilon_s$  and  $\epsilon_{\infty}$  are the limit of the permittivity at low and high frequencies,  $\tau$  is the relaxation time,  $\sigma_i$  is the ionic conductivity,  $\epsilon_0$  is the permittivity of free space and  $\alpha$  is a distribution parameter. For  $\alpha = 0$ , Equation 1 becomes the well-known Debye equation for a single

relaxation time process. All the parameters were floated except for the infinite permittivity value.

The data were fitted in turns to the Debye and Cole–Cole models. At concentrations below 0.5 mol/L, measurements over the entire frequency range (130 MHz–20 GHz) were used. At concentrations above 0.5 mol/L, a more limited data set (500 MHz–20 GHz) was used.

Figure 2a and b shows the measured permittivity and conductivity values together with the prediction of the Cole–Cole model; for clarity, not all concentrations measured are shown. The fitted parameters to the Debye and Cole–Cole models and the 95% confidence intervals associated with each parameter are shown in Tables 2 and 3 respectively. Included in these tables are the root mean square error (RMSE) values of each fit.

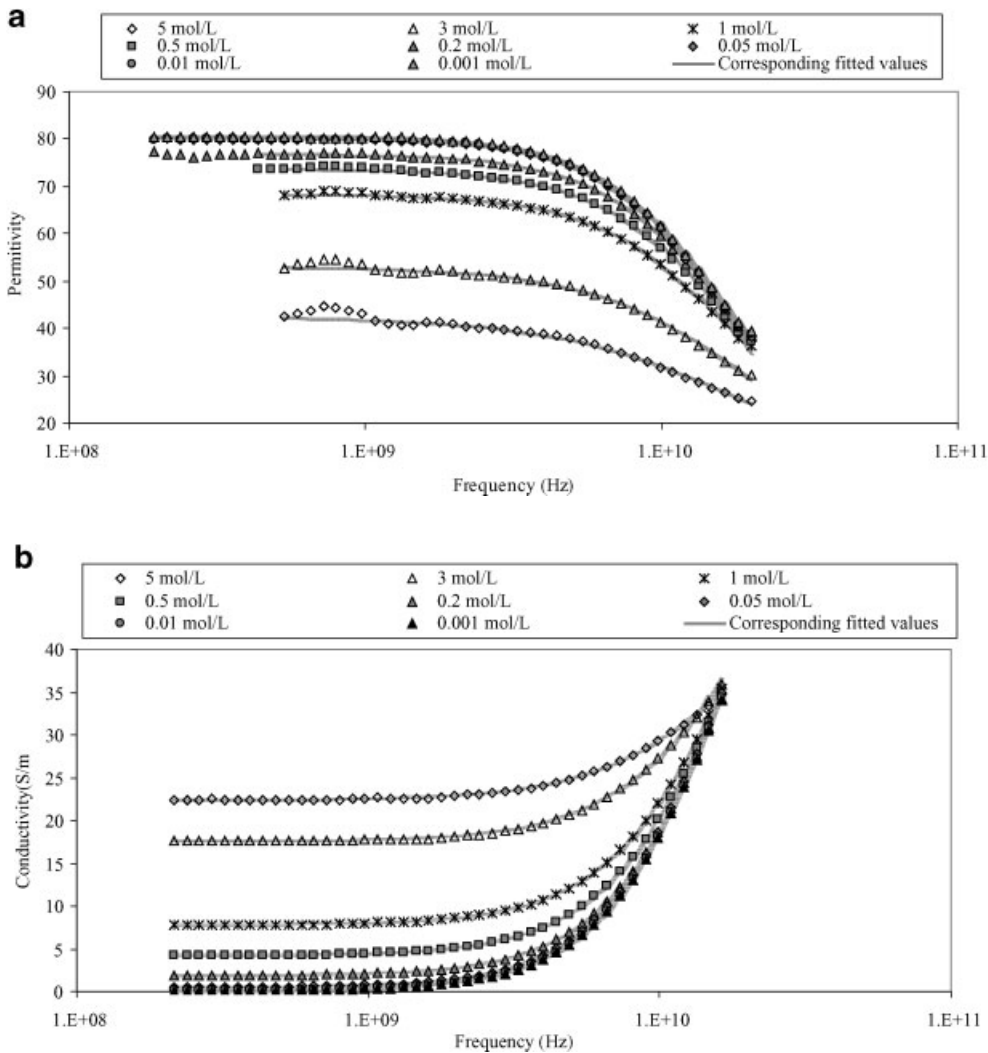


Fig. 2. The measured (a) permittivity and (b) conductivity of selected NaCl solutions and corresponding Cole–Cole fitted values.

**TABLE 2. Dielectric Parameters of Different NaCl Solutions Obtained by Fitting the Experimental Results at 20 °C to a Debye Model\***

Concentration (mol/L)	$\epsilon_s$	$\pm \epsilon_s$	$\tau$ (ps)	$\pm \tau$ (ps)	$\sigma_i$ (S/m)	$\pm \sigma_i$	RMSE
0.001	80.5	0.1	9.18	0.08	0.01	0.00	0.38
0.005	79.9	0.1	9.22	0.08	0.05	0.00	0.36
0.01	80.1	0.1	9.26	0.08	0.11	0.00	0.37
0.05	80.1	0.1	9.38	0.11	0.49	0.00	0.54
0.07	78.8	0.2	9.13	0.15	0.72	0.00	0.73
0.09	78.3	0.2	9.19	0.19	0.94	0.00	0.87
0.1	78.1	0.2	9.10	0.20	0.96	0.00	0.93
0.5	73.5	0.2	8.92	0.20	4.19	0.01	0.84
0.7	71.8	0.3	8.79	0.25	5.67	0.01	1.05
0.9	69.1	0.3	8.52	0.32	7.09	0.02	1.32
1	67.9	0.3	8.53	0.33	7.81	0.02	1.35
2	59.4	0.5	8.13	0.62	13.29	0.03	2.24
3	51.9	0.7	7.95	0.93	17.63	0.04	2.93
4	46.0	0.8	7.86	1.24	20.63	0.04	3.43
5	40.7	0.9	7.97	1.60	22.47	0.05	3.80

\*The term  $\pm$  corresponds to the 95% confidence interval.

At low concentrations ( $c < 0.5$  mol/L) the RMSE for the Debye and Cole–Cole fits are comparable and, in the case of Cole–Cole, the value for  $\alpha$  is indistinguishable from zero. This indicates that the Debye is the most appropriate model. At higher concentrations we obtain nonzero  $\alpha$  values and better fits to the Cole–Cole model.

Another interesting observation from the current data is that the dielectric parameters vary monotonically

with concentration up to 4 mol/L, beyond this the trend changes slightly and in the case of relaxation time it tends to reverse. This is suggestive of a saturation percolation effect, which, when present, is known to affect the dielectric properties [Sihvola, 1993; Tinga, 1992].

For comparison reasons, Figure 3a and b shows the measured permittivity and conductivity of selected

**TABLE 3. Dielectric Parameters of Different NaCl Solutions Obtained by Fitting the Experimental Results at 20 °C to a Cole–Cole Model**

Concentration (mol/L)	$\epsilon_s$	$\pm \epsilon_s^*$	$\tau$ (ps)	$\pm \tau^*$ (ps)	$\alpha$	$\pm \alpha^*$	$\sigma_i$ (S/m)	$\pm \sigma_i^*$	RMSE
0.001	80.6	0.1	9.16	0.08	0.005	0.003	0.01	0.00	0.36
0.005	80.0	0.1	9.21	0.08	0.002	0.003	0.05	0.00	0.36
0.01	80.1	0.1	9.25	0.08	0.001	0.003	0.11	0.00	0.36
0.03	80.2	0.1	9.37	0.10	0.001	0.003	0.30	0.00	0.44
0.05	80.1	0.1	9.37	0.12	0.002	0.004	0.49	0.00	0.54
0.07	78.9	0.2	9.12	0.16	0.005	0.006	0.72	0.00	0.72
0.09	78.4	0.2	9.18	0.19	0.005	0.007	0.94	0.00	0.86
0.1	78.2	0.2	9.08	0.20	0.006	0.008	0.96	0.00	0.92
0.2	77.0	0.2	8.98	0.10	0.009	0.004	1.83	0.01	0.44
0.5	74.0	0.2	8.91	0.13	0.023	0.006	4.17	0.01	0.53
0.7	72.4	0.2	8.80	0.15	0.023	0.006	5.64	0.01	0.57
0.9	70.1	0.2	8.50	0.14	0.038	0.006	7.04	0.01	0.54
1	68.7	0.2	8.55	0.16	0.038	0.006	7.76	0.01	0.61
1.5	65.3	0.3	8.44	0.21	0.040	0.009	10.54	0.01	0.72
2	60.9	0.3	8.13	0.26	0.065	0.011	13.22	0.02	0.81
2.5	56.8	0.4	8.01	0.34	0.085	0.015	15.85	0.02	0.95
3	53.8	0.4	7.95	0.41	0.104	0.018	17.53	0.02	1.04
3.5	50.5	0.5	7.86	0.51	0.125	0.022	19.48	0.02	1.14
4	48.5	0.5	7.89	0.58	0.147	0.024	20.51	0.02	1.19
4.5	46.1	0.6	7.98	0.70	0.175	0.028	21.62	0.03	1.26
5	43.9	0.7	8.13	0.85	0.207	0.032	22.33	0.03	1.31

\*The term  $\pm$  corresponds to the 95% confidence interval.

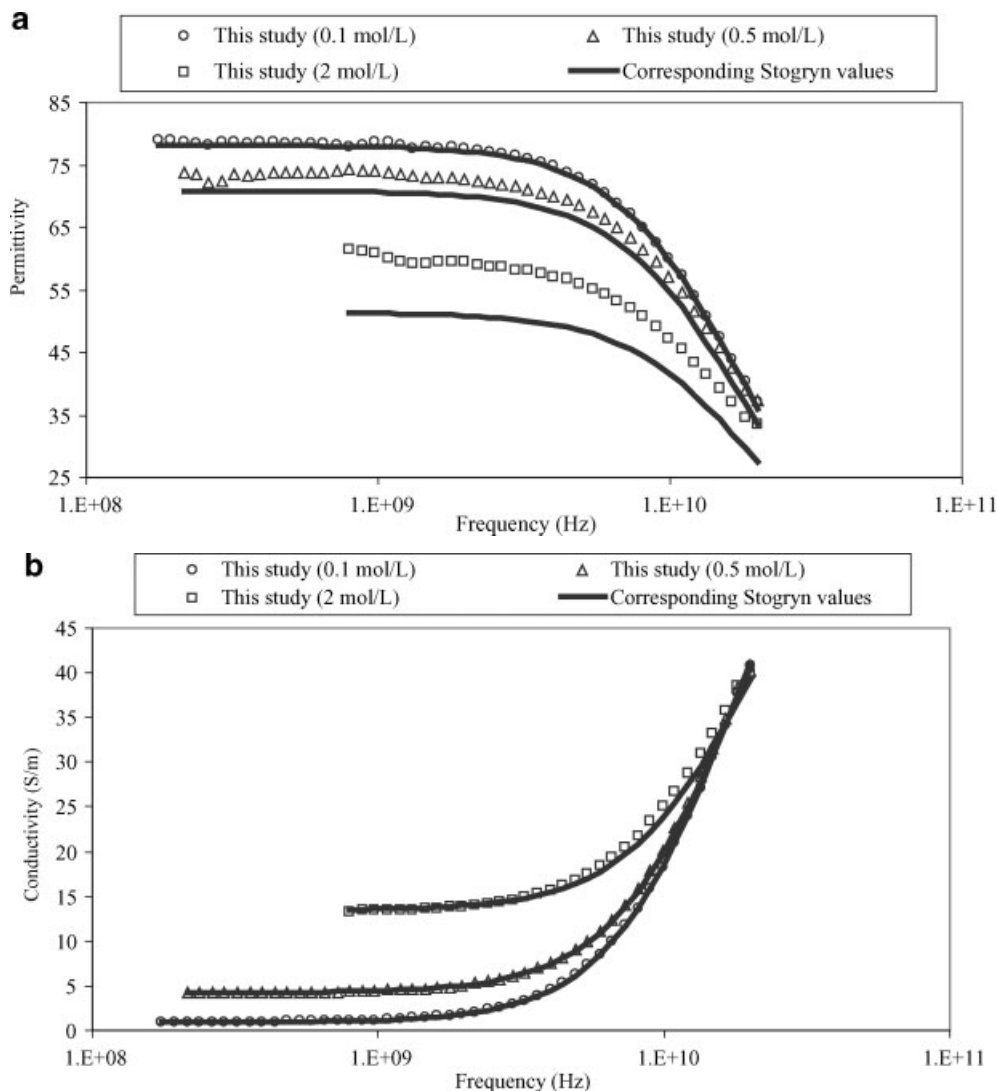


Fig. 3. Comparison between measured (a) permittivity and (b) conductivity of selected NaCl solutions compared to those calculated from Stogryn [1971] equations.

NaCl solutions at 20 °C together with those calculated using the Stogryn model. Tables 4 and 5 contain the dielectric parameters of NaCl solutions at 25 °C and 20 °C collated from different publications. A more complete account of the literature is given in Figures 4 and 5.

At low concentrations, where the Debye model applies, our data are in good agreement with the predictions of Stogryn's models. At higher concentrations the agreement breaks down for the reasons mentioned before.

The static permittivities agree (except at  $c = 5$  mol/L) with those reported by Hasted et al. [1948] which is as expected because Hasted's data suffer the same high concentration limitation as Stogryn's. Our results on the other hand show good agreement with more recent literature from Nortemann et al. [1997] and

Büchner et al. [1999] which together span the whole concentration range.

At low concentrations the relaxation times are in good agreement with Stogryn [1971], Lane and Saxon [1952], and Hasted et al. [1948]; increasingly large deviations occurs at higher concentration. As with the static permittivity, the agreement with the recent literature extends across the concentration range.

The ionic conductivity values agree well with data from standard conductivity tables (ELDAR) reported by Büchner et al. [1999] (see Fig. 4c).

It is clear (Fig. 4d) that the distribution factor ( $\alpha$ ) is indistinguishable from zero at low concentrations but increases gradually with concentration reaching nearly 0.2 at  $c = 5$  mol/L. A similar trend was observed by Büchner et al. [1999] and Harris and O'Konski [1957].

TABLE 4. Comparison of Literature Data for Dielectric Properties of NaCl Solutions at 25 °C

Concentration (mol/L)	Authors	$\epsilon_\infty$	$\epsilon_s$	$\tau$ (ps)	$\alpha$
0.1	Büchner et al. [1999]	5.65	76.6 ± 0.2	8.05 ± 0.03	0.008 ± 0.008
	Stogryn [1971]	4.9	76.3	8.09	—
	Hasted and Roderick [1958]	5 ± 0.3	78.0	8.17	0.025
0.5–0.7	Büchner et al. [1999] at 0.6 mol/L	5.65	70.5 ± 0.3	7.71 ± 0.01	0.026 ± 0.005
	Kaatze [1987] at 0.5 mol/L	4.4 ± 1	72.4	7.82	0.04
	Stogryn [1971] at 0.6	4.9	67.7	7.96	—
	Hasted and Roderick [1958] at 0.66 mol/L	5 ± 0.3	68.7	7.53	0.013
	Haggis et al. [1952] at 0.66 mol/L	5.5 ± 1	67.2	7.53	—
1	Büchner et al. [1999]	5.65	66.6 ± 0.2	7.47 ± 0.05	0.041 ± 0.005
	Stogryn [1971]	4.9	61.8	7.80	—
	Giese et al. [1970]	—	67.3	—	—
	Hasted and Roderick [1958]	5 ± 0.3	65.0	7.37	0.017
	Haggis et al. [1952]	5.5 ± 1	63.0	7.27	—
5	Büchner et al. [1999] at 4.5 mol/L	5.65	44.1 ± 0.6	6.50 ± 0.21	0.198 ± 0.002
	Pottel et al. [1989]	—	38.7	9.26	—
	Harris and O'Konski [1978]	—	42.0	7.00	0.18

### EQUATIONS FOR CALCULATING THE DIELECTRIC PARAMETERS AS A FUNCTION OF TEMPERATURE AND CONCENTRATION

#### Model Spanning all Available Concentrations and Temperatures

Data obtained in this study as from the literature were used to derive equations from which the dielectric parameters of NaCl solutions can be calculated as a

function of temperature and concentration. The data set used in the model encompasses  $c = 0\text{--}5$  mol/L and  $5^\circ\text{C}$  to  $35^\circ\text{C}$  (see Table 6). The data point from Pottel et al. [1989] for 5 mol/L NaCl at  $25^\circ\text{C}$  was not included because it differs quite significantly from all others (see Table 4). We assume this to be either a misprint in the paper or a misinterpretation on our part of this data point that is part of a study on the effect of high pressure on the dielectric response. The dielectric values for water taken from Kaatze [1989] were also added to the pooled

TABLE 5. Comparison of Literature Data for Dielectric Properties of NaCl Solutions at 20 °C

Concentration (mol/L)	Authors	$\epsilon_\infty$	$\epsilon_s$	$\tau$ (ps)	$\alpha$	$\sigma_i$ (S/m)
0.1	This study	5.0	78.2	9.08	0.006	0.96 ± 0.00
	Büchner et al. [1999] Interpolated between 0.09 and 0.13 mol/L	5.6	79.1	9.38	0.017	0.96
	Stogryn [1971]	4.9	78.1	9.25	—	0.92
0.5	This study	5.0	74.0	8.91	0.023	4.22 ± 0.02
	Büchner et al. [1999] Interpolated between 0.46 and 0.59 mol/L	5.6	74.0	8.97	0.03	4.33
	Nortemann et al. [1997]	5.6 ± 1	73.4 ± 0.7	8.7 ± 0.2	—	—
	Stogryn [1971]	4.9	70.8	9.12	—	4.23
	Hasted et al. [1948]*	—	73.9	9.07	—	—
1	This study	5.0	68.7	8.55	0.038	7.81(0.02)
	Büchner et al. [1999] at 0.9 mol/L	5.6	68.2 ± 0.1	8.64 ± 0.03	0.040 ± 0.003	7.77
	Stogryn [1971]	4.9	63.2	8.87	—	7.74
	Hasted et al. [1948]*	—	69.1	8.65	—	—
3	This study	5.0	53.8	7.95	0.104	17.57 ± 0.02
	Büchner et al. [1999] at 2.9 mol/L	5.6	52.7 ± 0.1	7.88 ± 0.20	0.103 ± 0.001	17.37
	Stogryn [1971]	4.9	41.0	7.67	—	17.75
	Hasted et al. [1948]*	—	54.0	7.80	—	—
5	This study	5.0	43.9	8.13	0.207	22.37 ± 0.03
	Büchner et al. [1999] at 4.5 mol/L	5.6	43.5 ± 0.3	7.61 ± 0.21	0.176 ± 0.004	21.69
	Hasted et al. [1948]*	—	48.4	7.37	—	—

\*Measurements performed at 21 °C.



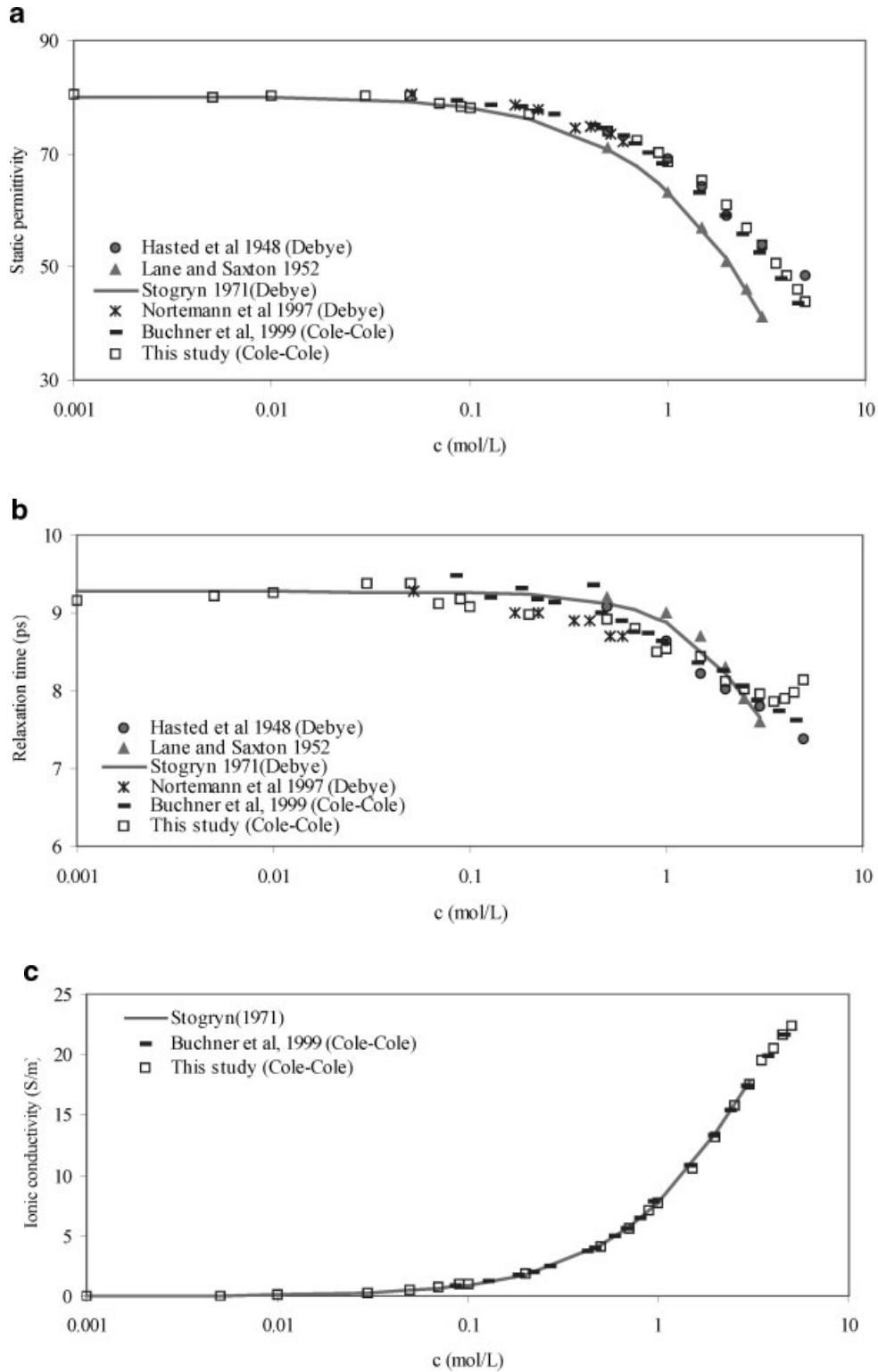


Fig. 4. Changes in dielectric parameters as a function of concentrations  $c$  (mol/L) for different NaCl solutions at 20 °C\* (a) static permittivity ( $\epsilon_s$ ), (b) relaxation time ( $\tau$ psec), (c) ionic conductivity ( $\sigma_i$ ) and (d) distribution parameter ( $\alpha$ ) \* Hasted et al. [1948] data were obtained at 21 °C.

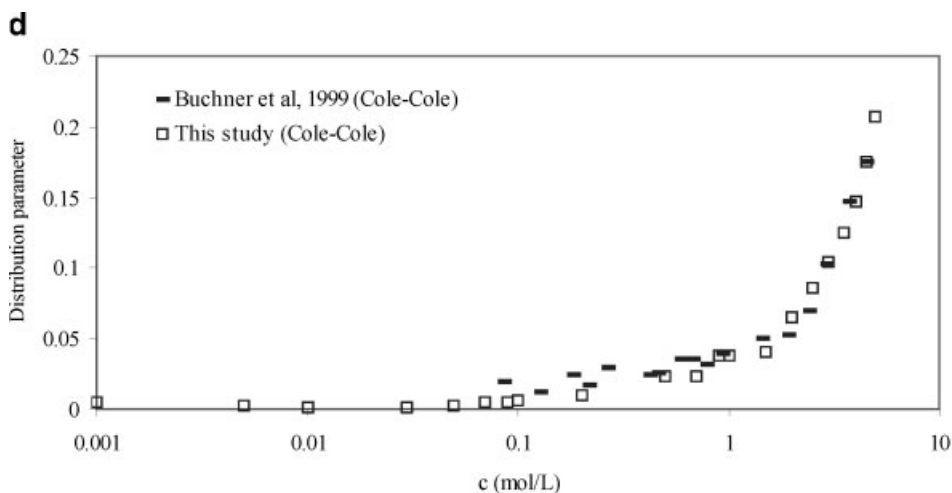


Fig. 4. (Continued)

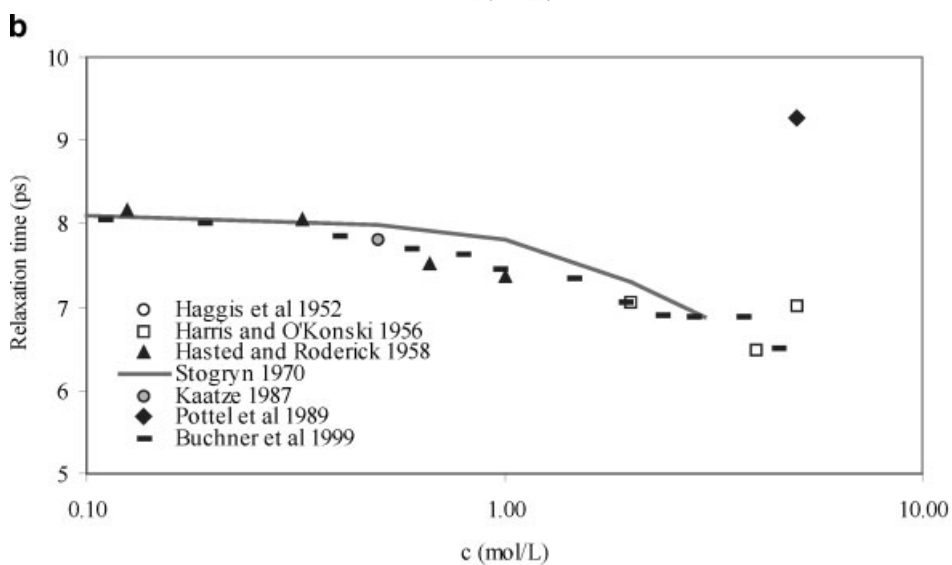
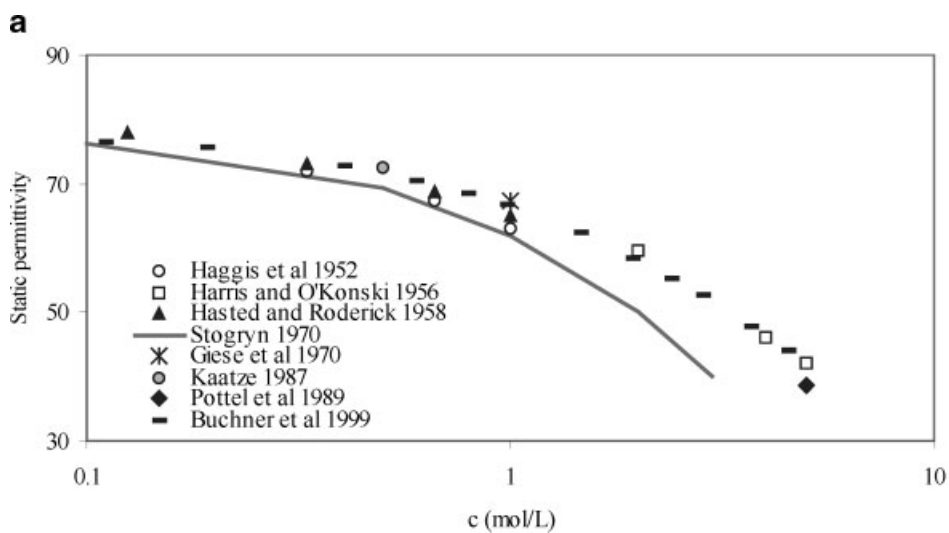


Fig. 5. Changes in dielectric parameters as a function of concentrations  $c$  (mol/L) for different NaCl solutions at 25 °C (a) static permittivity, (b) relaxation time.



**TABLE 6. The Sources of Data Used for Deriving the Polynomial Equations for Dielectric Properties of NaCl Solutions as a Function of Temperature and Concentration**

Source of data	Temperatures (°C)	Concentrations (mol/L)
Kaatze [1989]	0–35	0
This study	20	0.001–5
Büchner et al. [1999]	5–35	0.1–4.5
Nortemann et al. [1997]	20	0.05–0.6
Hasted et al. [1948]	21	0.5–5
Kaatze [1987]	25	0.5
Hasted and Roderick [1958]	25	0.1–1
Haggis et al. [1952]	25	0.3–1

data in order to take into account the concentration at  $c = 0$  mol/L.

Polynomial equations were derived from the data relating each of the static permittivity  $\varepsilon_s$ , relaxation time  $\tau$  (sec), ionic conductivity  $\sigma_i$  (S/m) and distribution parameter  $\alpha$  to the concentration and temperature of NaCl solutions in water. No model was derived for high frequency permittivity  $\varepsilon_\infty$  as no trend could be found as a function of concentration. Instead we propose the values for water as a function of temperature taken from Kaatze [1989] to be used for  $\varepsilon_\infty$

$$\varepsilon_\infty = 5.77 - 0.0274t \quad (2)$$

where  $t$  is the temperature in degrees Celsius.

The static permittivity  $\varepsilon_s$ , relaxation time  $\tau$  (sec), conductivity  $\sigma_i$  (S/m) and Distribution parameter  $\alpha$  for NaCl of 0–5 mol/L at 5°C–35 °C can be obtained from the following equations

$$\varepsilon_s = \varepsilon_s(w)(0.999 + 8.521 \times 10^{-4}tc + 0.013c^2 - 0.175c + 2.344 \times 10^{-4}t - 1.235 \times 10^{-5}t^2) \quad (3)$$

$$\tau(\text{sec}) = \tau(w)(1.03 + 9.387 \times 10^{-5}tc + 0.012c^2 - 0.091c - 3.093 \times 10^{-3}t + 4.932 \times 10^{-5}t^2) \quad (4)$$

$$\sigma_i = 0.096tc - 0.8c^2 + 6.554c \quad (5)$$

$$\alpha = 2.474 \times 10^{-4}tc + 2.101 \times 10^{-3}c^2 + 0.021c \quad (6)$$

where  $t$  is the temperature of the liquid in °C and  $c$  is the concentration of the solution in mol/L.  $\varepsilon_s(w)$  and  $\tau(w)$ (sec) are the static permittivity and relaxation time of water from Kaatze (1989)

$$\varepsilon_S(w) = 10^{(1.94404 - 1.991 \times 10^{-3}t)} \quad (7)$$

$$\tau(w) = 3.745 \times 10^{-15}(1 + (7 \times 10^{-5})(t - 27.5)^2) \exp(2.2957 \times 10^3/(t + 273.15)) \quad (8)$$

The accuracy of the above polynomials when calculating the dielectric properties of water at temperatures between 5°C and 35 °C are within 0%–0.7% for static permittivity and 0%–1.8% for relaxation time.

Equation 3 predicts the static permittivity of different concentrations and temperatures to within 0%–3% for most of the literature used. The static permittivity reported by Hasted and Roderick [1958] and Haggis et al. [1952] for  $c = 0.1$ – $0.6$  mol/L at 25 °C was predicted by this equation to within 1.7%–5% increasing to 6.5% at  $c = 1$  mol/L. Equation 4 falls within 0.1%–4% of the relaxation time data set.

Higher deviations from the model were observed for the ionic conductivity ( $\sigma_i$ ) and distribution factor ( $\alpha$ ) values.

### Model for Concentrations Below 1 mol/L

While fairly representative of the data, Equations (3)–(6) may not be sufficiently sensitive at low concentration because of the small magnitude and large dynamic range of some of the parameters. Models were therefore derived to accommodate low and high concentrations with an arbitrary breakpoint around 1 mol/L. At low concentrations of  $c = 0$ – $1$  mol/L at 5°C–35 °C we obtained

$$\varepsilon_s = \varepsilon_s(w)(1.0 - 3.742 \times 10^{-4}tc + 0.034c^2 - 0.178c + 1.515 \times 10^{-4}t - 4.929 \times 10^{-6}t^2) \quad (9)$$

$$\tau(\text{sec}) = \tau(w)(1.012 - 5.282 \times 10^{-3}tc + 0.032c^2 - 0.01c - 1.724 \times 10^{-3}t + 3.766 \times 10^{-5}t^2) \quad (10)$$

$$\sigma_i = 0.174tc - 1.582c^2 + 5.923c \quad (11)$$

$$\alpha = -6.348 \times 10^{-4}tc - 5.1 \times 10^{-2}c^2 + 9 \times 10^{-2}c \quad (12)$$

The accuracy of the above polynomials when calculating the dielectric properties of water at temperatures between 5°C and 35 °C are within 0%–0.1% for static permittivity and 0%–0.7% for relaxation time. These values are well-below the percentage error reported by Kaatze [1989] for dielectric parameters of water in the same temperature range (0.2%–0.3% for static permittivity and 0.7%–0.9% for relaxation time).

TABLE 7. The %Difference Between the Predicted and Original Values Using Three Polynomials for Different Concentrations of NaCl at Selected Temperatures

mol/L	% difference between the experimental and predicted data using Equations 3-6			% difference between the experimental and predicted data using Equations 9-12			% difference between the experimental and predicted data using Equations 13-16					
	$\epsilon_s$	$\tau$ (ps)	$\sigma_i$ (S/m)	$\alpha$	$\epsilon_s$	$\tau$ (ps)	$\sigma_i$ (S/m)	$\alpha$	$\epsilon_s$	$\tau$ (ps)	$\sigma_i$ (S/m)	$\alpha$
0.06* at 18 °C	-0.5	-0.2	-16.3	—	-0.4	0.0	-5.6	-66.7	—	—	—	—
0.06* at 20 °C	0.0	0.5	-13.7	—	0.0	0.9	-3.6	63.6	—	—	—	—
0.06* at 22 °C	0.6	1.0	-15.4	—	0.8	1.4	-3.4	91.5	—	—	—	—
0.06* at 24 °C	1.0	-0.1	-18.9	—	1.2	0.5	-5.0	95.7	—	—	—	—
0.13** at 20 °C	0.1	-0.5	12.4	—	0.3	-0.7	3.3	25.0	—	—	—	—
0.46**	-0.1	0.2	5.5	53.8	0.5	0.7	-1.0	3.8	—	—	—	—
0.9**	-1.3	-0.2	6.7	35.0	-0.7	0.0	4.4	30.0	-0.7	-1.3	5.9	2.5
1.0*	0.3	-0.8	1.2	26.3	0.9	-0.6	-0.8	31.6	0.9	-1.9	0.6	-7.9
1.9*** at 5 °C	-0.3	2.0	-7.1	5.6	—	—	—	—	0.0	0.0	-3.3	9.3
1.9** at 20 °C	-0.8	1.6	-1.3	-9.4	—	—	—	—	-1.7	0.5	1.1	-22.6
1.9** at 25 °C	-0.5	-0.4	2.0	-7.0	—	—	—	—	-1.2	0.6	1.8	-14.0
1.9** at 35 °C	0.0	0.4	7.9	-25.5	—	—	—	—	1.2	2.7	0.5	-3.9
4.54** at 20 °C	-1.4	-3.3	-1.3	8.5	—	—	—	—	-2.5	-1.3	-1.0	0.6
5.0* at 20 °C	4.6	2.0	-0.2	12.1	—	—	—	—	2.3	4.8	-0.9	2.9

\*The experimental data were obtained in this study (the 0.06 mol/L were used as a blind test).

\*\*The experimental data were obtained from Büchner et al. [1999].

Equation 9 predicts the static permittivity of different concentrations and temperatures to within 0.1%–1% for most of the literature used. Equation 10 predicts the relaxation time of the pooled data to within 0.1%–3%. The static permittivity and relaxation time reported by Hasted and Roderick [1958] and Haggis et al. [1952] for  $c=0.1-0.6$  mol/L at 25 °C were predicted by this equation to within 1.2%–4.0% and 0.6%–3% respectively.

As expected, the conductivity and distribution factor ( $\alpha$ ) are better predicted by (11) and (12) as opposed to (5) and (6) (see Table 7).

**Model for Concentrations of 1 to 5 mol/L**

The high concentration model produced

$$\epsilon_s = 84.328 + 0.117tc + 0.77c^2 - 13.257c - 0.207t - 4.859 \times 10^{-3}t^2 \quad (13)$$

$$\tau(ps) = 17.76 + 0.022tc + 0.09c^2 - 1.222c - 0.525t + 5.361 \times 10^{-3}t^2 \quad (14)$$

$$\sigma_i = 0.061tc - 0.667c^2 + 6.485c - 0.439 - 1.2 \times 10^{-2}t + 3.374 \times 10^{-3}t^2 \quad (15)$$

$$\alpha = 0.011 + 4.326 \times 10^{-4}tc + 4.431 \times 10^{-3}c^2 + 4.754 \times 10^{-3}c + 1.82 \times 10^{-3}t - 6.154 \times 10^{-5}t^2 \quad (16)$$

Equation 13 predicts the static permittivity to within 0%–2% for most of the literature used. Equation 14 predicts the relaxation time for most of the literature well-below 2%.

The static permittivity and relaxation time reported by Hasted and Roderick [1958] and Haggis et al. [1952] for  $c = 1-5$  mol/L NaCl at 25 °C were predicted by these equations to within 1%–5% and 0%–4% respectively.

**Validation of the Polynomial Equations**

The predictions of the models are compared in Table 7. The measured dielectric values for  $c = 0.06$  mol/L NaCl were used as a blind test and were not included in the data set from which the models were derived.

It can be seen from Table 7 that at concentrations below 1 mol/L, the percentage differences between the predicted and original values are quite comparable for static permittivity and relaxation time values when either set of polynomials are used. However, the errors

in the predicted conductivity and distribution factor ( $\alpha$ ) values are reduced notably when the low concentration model is applied. A similar trend, but to a lesser extent, can be observed at concentrations above 1 mol/L.

## SUMMARY

The dielectric properties of different concentrations of NaCl solutions at 20 °C have been measured and the results agree well with the relatively recent literature. Agreement with Stogryn [1971] was limited to concentrations below 0.5 mol/L.

Both Debye and Cole–Cole models were used to describe the dielectric behaviour of NaCl solutions. We found that the Debye expression is a better model for low concentration NaCl solutions ( $c < 0.5$  mol/L), the while Cole–Cole model becomes more appropriate at higher concentrations ( $c > 0.5$  mol/L).

The experimental results obtained in this study and those reported in the literature were combined to derive polynomial models that predict the dielectric behaviour of NaCl solutions between  $c = 0$ –5 mol/L and 5°–35 °C. More accurate predictions are obtained for ionic conductivity ( $\sigma_i$ ) and distribution factor ( $\alpha$ ) when separate models are used for concentrations below and above 1 mol/L.

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